sensors and an intersatellite ranging system) with the processed ground tracking information, a good linear estimate of these three parameters can be obtained. Care must be taken in defining the variables to be estimated to insure a low error in the final output.

Appendix

The state transition matrix $\Phi(t,t_0)$ is

$$\Phi(t,t_0) = \left[\begin{array}{c|c} \Phi_1(t,t_0) & 0 \\ \hline 0 & \Phi_2(t,t_0) \end{array}\right] \tag{A1}$$

$$\Phi_1(t,t_0) = \begin{bmatrix} (4 - 3\cos\omega_0\tau) & 0 & (\sin\omega_0\tau)/\omega_0 & 2(1 - \cos\omega_0\tau)/\omega_0 \\ 6(\sin\omega_0\tau - \omega_0\tau) & 1 & 2(\cos\omega_0\tau - 1)/\omega_0 & (4\sin\omega_0\tau - 3\omega_0\tau)/\omega_0 \\ 3\omega_0\sin\omega_0\tau & 0 & \cos\omega_0\tau & 2\sin\omega_0\tau \\ 6\omega_0(\cos\omega_0\tau - 1) & 0 & -2\sin\omega_0\tau & (4\cos\omega_0\tau - 3) \end{bmatrix} \tag{A2}$$

$$\Phi_2(t,t_0) \begin{bmatrix} \cos\omega_0\tau & (\sin\omega_0\tau)/\omega_0 \\ -\omega_0\sin\omega_0\tau & \cos\omega_0\tau \end{bmatrix} \tag{A3}$$

$$\tau = t - t_0 \tag{A4}$$

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Use of Venus Gravitational Field for Solar Probe Trajectory Control

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The possibility of use of braking capabilities of the Venus gravitational field for probe trajectory control is discussed. The control problem consists in minimizing the probe flight time from the launch moment up to the moment of probe eclipse by the sun. The Earth and Venus coordinates are computed over the mean Newcomb elements. The probe flight trajectory is considered to be Keplerian and radius of the Venus sphere action is held to be a negligible. On the basis of the analysis, main characteristics of the flight trajectories as well as their interrelations are determined. The efficiency of using the Venus gravitational field is evaluated according to the velocity required for placing the probe into the flight trajectory and for trajectory correction.

Nomenclature

- $a_\odot$ = semimajor axis of probe's heliocentric orbit after Venus flyby
- $a,r,\mathbf{V}$ = semimajor axis of Venus orbit, distance from Venus to the sun, and Venus velocity vector at the moment of probe's impact of Venus, respectively
- R = Venus radius
- $t_\odot$ = probe transfer time from the Earth to the moment of probe eclipse by the sun
- $t_♀$ = probe transfer time from the Earth to Venus
- $t_☊,t_☋$ = probe transfer time from the Earth to a probe orbit ascending and descending ecliptic nodes, respectively
- $t_{\oplus☊},t_{\oplus☋}$ = Earth transfer time from departure date to probe orbit ascending and descending ecliptic node, respectively
- $T_\odot,T_\oplus$ = sidereal and synodic periods of a probe's heliocentric orbit after Venus flyby, respectively
- $\mathrm{grad}_1 T_\odot,\mathrm{grad}_2 T_\odot$ = gradients of probe orbit period at the moments of both the first and the second corrections in probe velocity at the same moments, respectively
- $\mathrm{grad}_1 t_☊$ = gradient of probe flight time from the first correction moment up to the moment of the eclipse node passing in the probe velocity at the first correction moment
- $V_♀$ = velocity of a probe with respect to Venus upon the sphere of its action
- $\mathbf{V}_{♀1},\mathbf{V}_{♀2}$ = vectors of probe velocity with respect to Venus upon the sphere of its action before and after Venus flyby, respectively
- $V_\oplus$ = velocity of a probe with respect to the Earth upon the sphere of its action
- V_ξ,V_η = projections of Venus velocity vector along $O\xi$ and $O\eta$ axes, respectively
- $\Delta\rho$ = maximum deviation of a point of impact from an aiming point on target plane

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$\mu_\odot, \mu_♀$ = gravitational parameters of the sun and Venus, respectively
ξ, η = coordinates of point of impact of probe on target plane
$\boldsymbol{\xi}^\circ, \boldsymbol{\eta}^\circ, \boldsymbol{\zeta}^\circ$ = unit vectors of Cartesian coordinates; $O\xi\eta\zeta$ coordinates origin is a center of mass of Venus; axis $O\zeta$ is directed against vector $\mathbf{V}_{♀1}$, i.e., the coordinate plane $O\xi\eta$ is a target plane; axis $O\xi$ is directed along a line of intersection of a target plane with Venus orbital plane out of the Sun; axis $O\eta$ is directed along a projection of a Venus orbit angular momentum on target plane
$\rho_♀$ = radius of peridistance of probe's Venus flyby trajectory
τ = Earth departure date

Introduction

THIS paper deals with probe flight trajectories that result in probe eclipse by the sun as observed from the Earth. The Venus flyby maneuver is used to reduce outbound flight time to the moment that eclipse begins.[1] At the outer Venus flyby relative to the sun, the disturbing effect of Venus' gravitational field leads to the probe absolute velocity decrease after flyby[2,3] and consequently to the decrease of side-real period $T_\odot$ of the probe's heliocentric orbit. Together with the $T_\odot$ decrease, much more noticeable decreases of synodic period $T_\oplus$ and outbound transfer time $t_\odot$ occur.

In Figs. 1 and 2, dependences of $T_\odot$, $T_\oplus$, and $t_\odot$ on velocity $V_\oplus$ at different radii of peridistance of the probe's Venus flyby trajectory $\rho_♀$ are shown. It is seen that $T_\odot$ and $t_\odot$ are 0.72 yr and 1.12 yr, respectively, without performing the flyby maneuver ($\rho_♀ = \infty$) when the probe has a trajectory close to an optimal trajectory of flight to Venus ($V_\oplus = 4$ km/sec). By making the Venus flyby maneuver when $\rho_♀ = 2R$, $T_\odot$ can be reduced to 0.49 yr, and $t_\odot$ to 0.6 yr. To obtain the same transfer time without the flyby maneuver, the probe velocity must be 9 km/sec.

The curves of Figs. 1 and 2 are computed for the case of circular coplanar orbits of Venus and the Earth to represent the capabilities of use of the Venus-gravitational-field-braking effect for the given problem. In the case of noncoplanar orbits the problem is complicated due to the fact that probe eclipse can be carried out only on the line of probe orbit ecliptic nodes. In addition, the influence of Venus flyby accuracy on the correction velocity after Venus flyby is essential for the given problem.

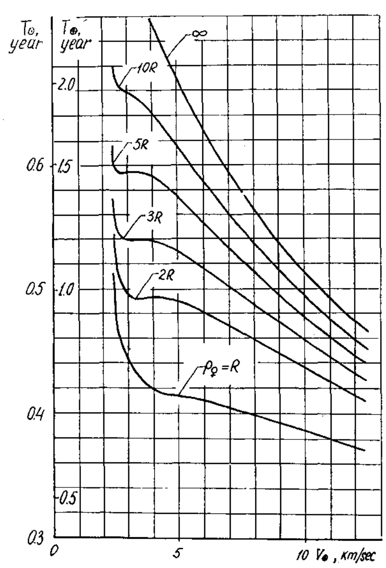

Fig. 1 Sidereal period $T_\odot$ and synodic period $T_\oplus$ vs probe velocity $V_\oplus$ with respect to the Earth upon the sphere of its action for different radii of peridistance of the Venus flyby trajectory $\rho_♀$.

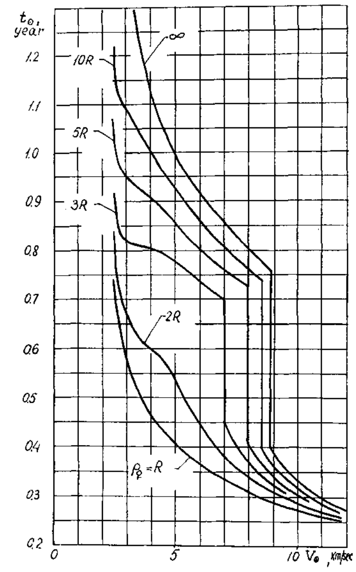

Fig. 2 Probe transfer time $t_\odot$ vs $V_\oplus$ for different values of $\rho_♀$.

Definition of Reference Trajectories

According to the formulated problem the probe reference trajectory must satisfy four conditions

1) $$V_\oplus(\tau, t_♀, \xi, \eta) \leq 5 \text{ km/sec} \tag{1}$$

2) $$\rho_♀(\tau, t_♀, \xi, \eta) > R \tag{2}$$

3) Because of noncoplanarity of the Earth's and probe's orbits, the probe as well as the Earth at the eclipse moment must be on the line of probe orbit nodes at opposite sides relative to the sun.

Therefore, for the probe eclipse on the ascending (or descending) node of its orbit it is necessary that the probe transfer time up to the ascending (or descending) node of its orbit t_Ω (or $t_\mho$) be equal to the Earth transfer time up to the descending (respectively, ascending) node of the probe orbit $t_{\oplus\mho}$ (respectively, $t_{\oplus\Omega}$)

$$t_\Omega(\tau, t_♀, \xi, \eta) = t_{\oplus\mho}(\tau, t_♀, \xi, \eta)$$
$$[\text{or } t_\mho(\tau, t_♀, \xi, \eta) = t_{\oplus\Omega}(\tau, t_♀, \xi, \eta)] \tag{3}$$

and

4) $$T_\odot(\tau, t_♀, \xi, \eta) = 0.5 \text{ yr} \tag{4}$$

In this case the synodic period $T_\oplus$ amounts to 1 yr, and the re-eclipses can be observed each year after performing the first eclipse.

The reference trajectory satisfying conditions 1–4 is determined under the assumption that the probe motion is Keplerian. The radii of the spheres of action of the Earth and Venus are considered to be negligible. The flyby maneuver is computed under the assumption that transfer time to the Venus target plane is independent of coordinates of the point of impact on the target plane ξ, η. For the solution of Kepler's and Lambert's equations, Battin's universal formulas[4] are used. The coordinates and the velocity components of the Earth and Venus are computed over the mean Newcomb elements.[5]

With such a model of motion the probe trajectory is determined by four parameters: τ, $t_♀$, ξ, and η. Conditions 3 and 4 reduce the number of free parameters to two, and τ and $t_♀$ are chosen as free parameters. Conditions 1 and 2 define the domain of τ and $t_♀$ variations within which it is possible to get a reference trajectory. The boundary of the domain and the mutual dependences of reference trajectory char-

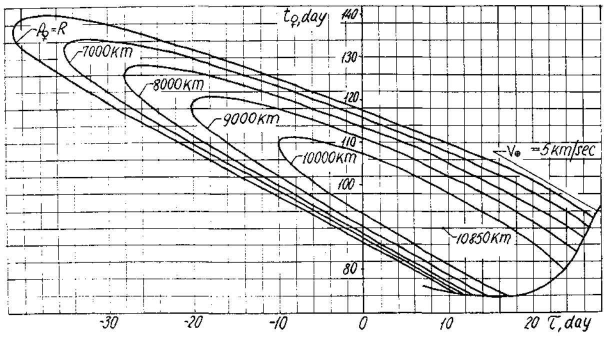

Fig. 3 Isolines of $\rho_♀$ on the plane of departure data τ (measured from 12:00 hr on June 15, 1975) vs time of flight to Venus $t_♀$.

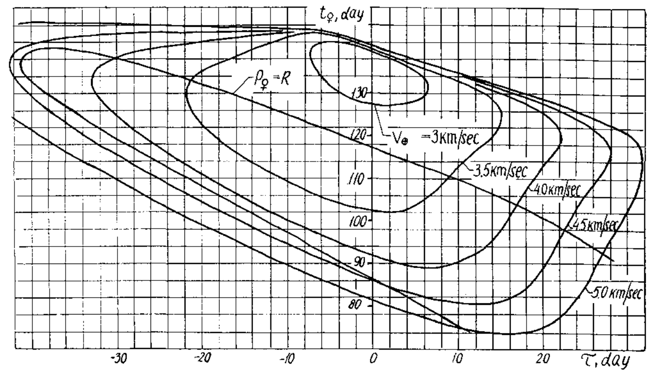

Fig. 5 Isolines of $V_\oplus$ on the plane of τ vs time of flight to Venus $t_♀$.

acteristics are computed as follows. First, the one-by-one analyses of flight trajectories are carried out for which parameters τ, $t_♀$, and $\rho_♀$ are within the ranges determined by conditions 1 and 2. Then, using the given values of τ, $t_♀$, and $\rho_♀$, Eq. (4) is solved relative to ξ and η by the following transformations. Let us write Eq. (4) as

$$-\mu_\odot/2a_\odot = (\mathbf{V} + \mathbf{V}_{♀2})^2/2 - \mu_\odot/r \tag{5}$$

where $a_\odot$ is a semimajor axis of probe orbit uniquely defined by the given $T_\odot$ of 0.5 yr. For the relative velocity $\mathbf{V}_{♀2}$, we can write

$$\mathbf{V}_{♀2} = -V_♀ (\cos\gamma \sin f \boldsymbol{\xi}^\circ + \sin\gamma \sin f \mathbf{n}^\circ + \cos f \boldsymbol{\zeta}^\circ) \tag{6}$$

where γ is the polar angle of the point of impact on the target plane

$$\cos\gamma = \xi/\rho,\ \sin\gamma = \eta/\rho,\ \rho = \rho_♀(1 + 2\mu_♀/\rho_♀ V_♀{}^2)^{1/2} \tag{7}$$

and f is the angle of rotation of the relative velocity vector during Venus flyby[6]

$$\sin(f/2) = 1/(1 + \rho_♀ V_♀{}^2/\mu_♀),\ 0 \leqslant f \leqslant \pi \tag{8}$$

Substituting Eq. (6) into Eq. (5) and solving the resulting equation with respect to γ, we obtain

$$\cos(\gamma - \beta) = (A + \cos f \cos\alpha)/\sin f \sin\alpha \tag{9}$$

where β is the polar angle of the Venus velocity vector projection on target plane

$$\cos\beta = V_\xi/(V_\xi{}^2 + V_\eta{}^2)^{1/2},\ \sin\beta = V_\eta(V_\xi{}^2 + V_\eta{}^2)^{1/2} \tag{10}$$

and α is the angle between $\mathbf{V}$ and the vector of the relative velocity before Venus flyby $\mathbf{V}_{♀1}$; the value A is

$$A = V_♀/2V(1 - \mu_\odot/aV_♀{}^2 + \mu_\odot/a_\odot V_♀{}^2) \tag{11}$$

From Eq. (9) it follows that if the value of A satisfies an inequality

$$-\cos(f - \alpha) < A < -\cos(f + \alpha) \tag{12}$$

then the two solutions for angle γ exist

$$\gamma_{1,2} = \pm\arccos[(A + \cos f \cos\alpha)/\sin f \sin\alpha] + \beta \tag{13}$$

of $A = -\cos(f + \alpha)$, then $\gamma = \beta$, and the accepted value $\rho_♀$ is the greatest one at which it is possible to obtain the given period. If $A = -\cos(f - \alpha)$, then $\gamma = \beta + \pi$, and this value $\rho_♀$ is the smallest one at which this period may be obtained. Thus, Eqs. (7) and (13) allow us to eliminate ξ and η from Eq. (3); Eq. (3) determines the dependence of $t_♀$ on τ and $\rho_♀$, which is computed numerically by the method of tangents.

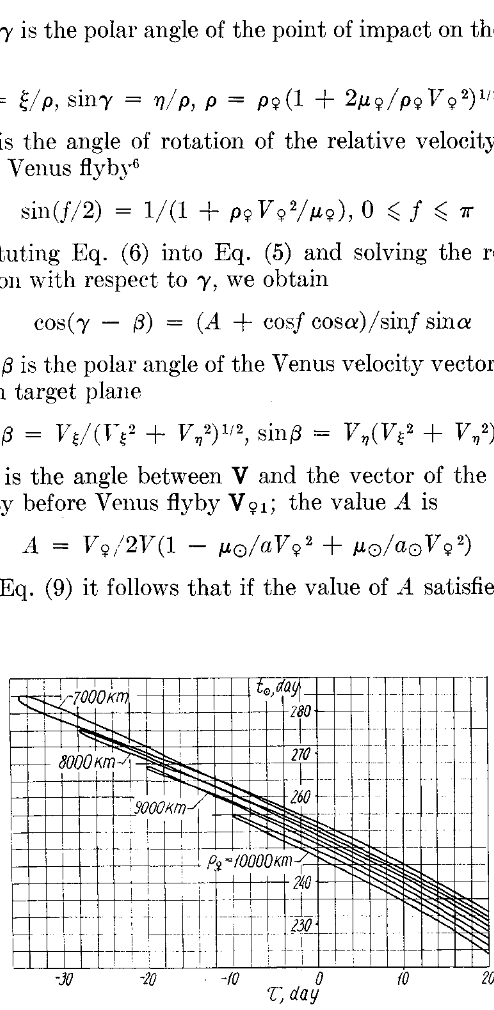

Fig. 4 Probe transfer time $t_\odot$ vs τ for different radii of Venus flyby trajectory peridistance $\rho_♀$.

Calculation of Trajectory Correction

The correction velocity $\mathbf{V}_K$ is determined for the disturbed orbit of a probe, which is obtained in terms of the deviations $\delta\xi$ and $\delta\eta$ of the coordinates of the probe point of impact on the Venus target plane, and $\delta\xi$ and $\delta\eta$ are limited by the inequality

$$(\delta\xi)^2 + (\delta\eta)^2 \leq (\Delta\rho)^2 \tag{14}$$

The influence on V_K of the deviation $\delta t_♀$ of the time of impact on the target plane from the reference value can be neglected. Actually, the effect of a $\delta t_♀$ of $\sim$3 days is equivalent to the effect of a $\Delta\rho$ of 1000 km.

The purpose of the correction after Venus flyby is to recover conditions 3 and 4. Now the nodes of the probe's disturbed orbit are considered in Eq. (3). It is obvious that $\mathbf{V}_K$ will always lie in the plane of the probe's disturbed orbit using such a choice of corrected parameters. Thus, with one-impulse correction, t_Ω and $T_\odot$ at the moment t_1 of application of the two-dimensional correction velocity vector $\mathbf{V}_{K_1}$ can be derived from the set of equations

$$\text{grad}_1 t_\Omega \cdot \mathbf{V}_{K1} = -\delta t_\Omega,\ \text{grad}_1 T_\odot \cdot \mathbf{V}_{K_1} = -\delta T_\odot \tag{15}$$

where δt_Ω is the difference between the probe transfer time (from the correction moment to the moment of reaching of corresponding node of disturbed orbit) and the Earth transfer time (from the correction moment to the moment of reaching of opposite node of probe disturbed orbit), and $\delta T_\odot$ is the difference between the disturbed period and 0.5 yr. The set Eq. (15), can turn out to be incompatible. For example, in case of $t_\Omega - t_1 = T_\odot$, the determinant Δ of Eq. (15) is equal to zero, and the one-impulse correction cannot be done. As a result, both one-impulse and two-impulse corrections are considered at the moments t_1 and t_2. At t_1, t_Ω is completely corrected and $T_\odot$ is partially corrected; at t_2, only $T_\odot$ is corrected. Evidently $\mathbf{V}_{K2}$ can be applied only at the moment the probe reaches the orbit node at which the eclipse must take place. The set of equations for defining both two-di-

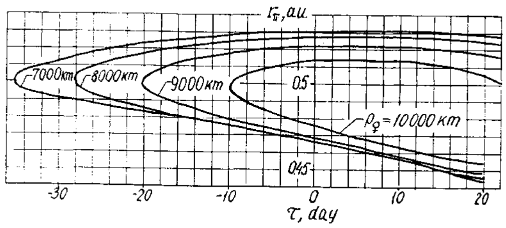

Fig. 6 Perihelion radius of probe orbit after Venus flyby r_π vs τ for different values of $\rho_♀$.

mensional vectors, $\mathbf{V}_{K_1}$ and $\mathbf{V}_{K_2}$, can be written

$$\text{grad}_1 t_\Omega \cdot \mathbf{V}_{K_1} = -\delta t_\Omega, \ \text{grad}_1 T_\odot \cdot \mathbf{V}_{K_1} + \text{grad}_2 T_\odot \cdot \mathbf{V}_{K_2} = -\delta T_\odot \quad (16)$$

The condition of minimization of the sum of V_{K_1} and V_{K_2} is additional to the set Eq. (16). It can be shown that taking into account this condition, the unique vectors $\mathbf{V}_{K_1}$ and $\mathbf{V}_{K_2}$ exist which satisfy the set, Eq. (16) and their values are

$$V_{K_1} = |\text{grad}_2 T_\odot||\delta t_\Omega|/(\text{grad}_1{}^2 t_\Omega \text{grad}_2{}^2 T_\odot - \Delta^2)^{1/2} \quad (17)$$

$$V_{K_2} = ||(\text{grad}_1 t_\Omega \cdot \text{grad}_1 T_\odot)\delta t_\Omega/\text{grad}_1{}^2 t_\Omega - \delta T_\odot| - \Delta^2|\delta t_\Omega|/\text{grad}_1{}^2 t_\Omega(\text{grad}_1{}^2 t_\Omega \cdot \text{grad}_2{}^2 T_\odot - \Delta^2)^{1/2}|/|\text{grad}_2 T_\odot| \quad (18)$$

where Δ is a determinant of the set, Eq. (15). The term of Eq. (18) with Δ^2 as a multiplier determines such a part of $\delta T_\odot$ that is expedient to correct at the first correction. If $\Delta = 0$, then only the two-impulse correction is possible. It follows from Eqs. (17) and (18) that in this case

$$V_{K_1} = |\delta t_\Omega|/|\text{grad}_1 t_\Omega| \quad (19)$$

$$V_{K_2} = |(\text{grad}_1 t_\Omega \cdot \text{grad}_1 T_\odot)\delta t_\Omega/\text{grad}_1{}^2 t_\Omega - \delta T_\odot|/|\text{grad}_2 T_\odot| \quad (20)$$

As Eqs. (19) and (20) indicate, the two one-parameter corrections are carried out here. If $\Delta \neq 0$, then both correction schemes are possible. A scheme giving a lower required velocity of correction V_K has been chosen in this case for each disturbed orbit. For the one-impulse correction, $V_K = V_{K_1}$, and for the two-impulse correction, $V_K = V_{K_1} + V_{K_2}$.

For given t_1 and $\Delta\rho$, the value $V_{K_{\max}}$ that is the greatest value of V_K at variations $\delta\xi$ and $\delta\eta$ within the domain, Eq. (14) is

$$V_{K_{\max}}(t_1,\Delta\rho) = \max_{\delta\xi,\delta\eta} V_K(t_1,\delta\xi,\delta\eta) \quad (21)$$

Finally, we determine the optimal value of t_1, $t_{1_{\text{opt}}}$, that minimizes $V_{K_{\max}}$ within the interval of possible $t'_1{}^s$

$$t_♀ + \Delta t_d \leq t_1 \leq t_\Omega \quad (22)$$

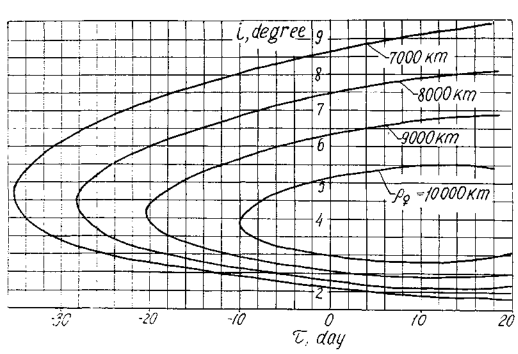

Fig. 7 Probe orbit inclination to the ecliptic plane after Venus flyby i vs τ for different values of $\rho_♀$.

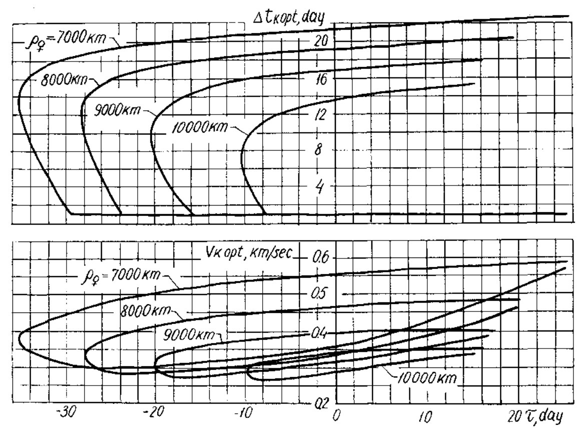

Fig. 8 Optimal correction velocity $V_{K\text{opt}}$ and optimal correction time $\Delta t_{1\text{opt}}$ after the moment of Venus flyby vs τ for different values of $\rho_♀$.

where Δt_d is a time required for the probe's disturbed orbit determination. The $V_{K_{\max}}$ corresponding to $t_{1_{\text{opt}}}$ defines the required $V_{K_{\text{opt}}}$ for given values of Δt_d and $\Delta\rho$

$$V_{K_{\text{opt}}}(\Delta t_d,\Delta\rho) = \min_{t_1} V_{K_{\max}}(t_1,\Delta\rho) \quad (23)$$

Results of Calculations

All calculations are performed for the year 1975, and a Universal Time of 12 hr on June 15 corresponds to the Earth departure date τ equal to zero.

Only the first-leg transfer to Venus is considered. For all considered dates the probe passes at first the ascending node of its orbit after Venus flyby. As it has been found it is impossible to accomplish the eclipse on this node, as the value $V_\oplus$ is limited by 5 km/sec. Thus the probe eclipse occurs only on the descending node.

Figure 3 presents the isolines of peridistance radius $\rho_♀$ in the plane τ and $t_♀$. The isosoline $V_\oplus = 5$ km/sec also is shown. Isolines $\rho_♀ = R$ and $V_\oplus = 5$ km/sec confine the domain of τ and $t_♀$ variations within which it is possible to obtain a reference trajectory. This domain corresponds to the first solution of Eq. (13), $\gamma = \gamma_1$. When $\gamma = \gamma_2$ the domain confined by $\rho_♀ = R$ is beyond the domain confined by $V_\oplus = 5$ km/sec. In Venus flyby a probe passes outside of the sun, crossing Venus' solar shadow. Venus flyby can be performed passing Venus inside the sun. Such trajectories also can be obtained at $\gamma = \gamma_1$ in the case of the second-leg-transfer are considered.

As seen from Fig. 3, two reference trajectories exist for a given set τ, $\rho_♀$; one corresponds to the greater transfer time $t_♀$ ("slow" trajectory), the other to the smaller transfer time

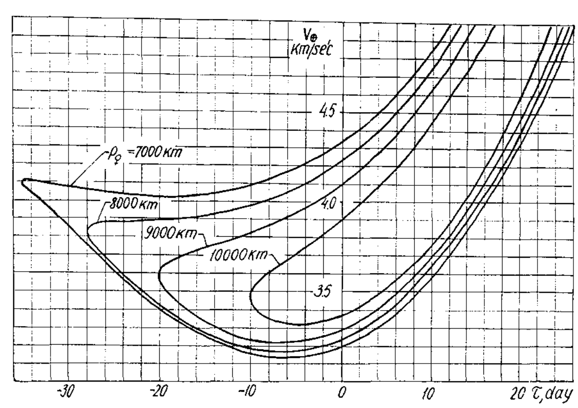

Fig. 9 Probe velocity $V_\oplus$ vs τ for different values of $\rho_♀$.

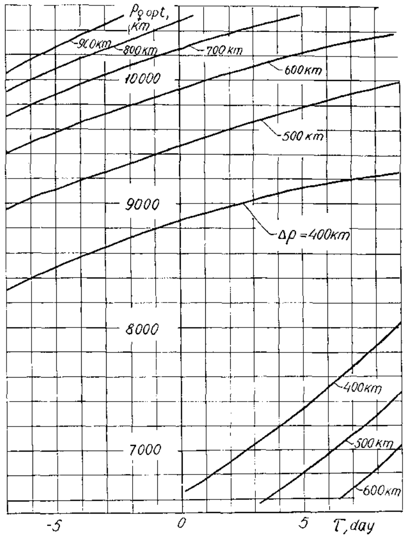

Fig. 10 Optimal radius of Venus flyby trajectory peridistance $\rho_{♀\,\mathrm{opt}}$ vs τ for different maximum declinations $\Delta\rho$ from the point of impact on the Venus target plane.

$t_♀$ ("fast" trajectory). The maximum value of $\rho_♀$, for reference trajectories is 10,850 km. A minimum value of $\rho_♀$ also exists, which is less than Venus radius for the given value $T_\odot$ of 0.5 yr. In general, the surface $\rho_♀(\tau_1 t_♀)$ is closed, similar to a deformed ellipsoid. It should be noted that the total transfer time to the eclipse $t_\odot$ for the "slow" and "fast" trajectories is much the same at a given set τ, $\rho_♀$. As seen from Fig. 4, $t_\odot$ is reduced with the increase of τ, so that for a given $\rho_♀$, the probe eclipse occurs on about the same date.

Figure 5 represents the solution domain relative to the isolines of $V_\oplus$. A minimum velocity $V_\oplus$ for the "slow" trajectories exceeds a minimum velocity for Venus flight by about 0.3 km/sec.

Figures 6 and 7 give the dependencies of perihelion radius $r\pi$ and probe orbit inclination i to the ecliptic plane after Venus flyby. The upper and the lower branches of the dependencies i and $r\pi$, respectively, correspond to the "fast" trajectories. According to condition 4, $a_\odot \approx 0.629$ a.u. for the probe orbit after flyby.

Figure 8 represents the dependencies of $V_{K_{\mathrm{opt}}}$ and $\Delta t_{1_{\mathrm{opt}}} = t_{1_{\mathrm{opt}}} - t_♀$ computed for a maximum declination value in the target plane $\Delta\rho$ of 1000 km and a time of trajectory determination after Venus flyby Δt_d of 1 day. The lower branches of the curves correspond to the "slow" trajectories. The optimal correction time for "fast" trajectories occurs about 10 to 20 days after Venus flyby. The "slow" trajectories must be corrected immediately after the flyby. The optimal values of radius of peridistance of flyby trajectory depending on $\Delta\rho$ exist for "slow" trajectories. Actually, as $\rho_♀$ is increased $V_\oplus$ is also increased (Fig. 9) and $V_{K_{\mathrm{opt}}}$ is decreased (Fig. 8). At the optimal value $\rho_♀$, the sum of $V_{K_{\mathrm{opt}}}$ and the velocity required for entering a reference trajectory by launch from a 200-km-altitude Earth orbit is minimized. Figure 10 shows the dependence of $\rho_{♀_{\mathrm{opt}}}$ on τ and $\Delta\rho$.

To reduce $V_{K\mathrm{opt}}$, one can give up the correction of probe orbit period. In this case, the $V_{K_{\mathrm{opt}}}$ correction velocity of "slow" trajectories is reduced by $\sim$50%. In addition, one can relax from providing the precise probe eclipse by the sun at which the angular distance of probe from the solar visible disk center is equal to zero. In this case, at the correction it is necessary to obtain the given minimum angular distance of the probe from the solar center.

Conclusions

From these calculations of the probe reference trajectories, the following conclusions can be drawn.

1) Use of the Venus gravitational field allows a reduction in probe transfer time from 1.2 to 0.6 yr. In this case, the probe velocity relative to the Earth within the sphere of its action exceeds the optimal velocity for flight to Venus by approximately 0.3 km/sec.

2) The repetition of probe eclipse every year after the first eclipse is possible. The maximum peridistance radius of the Venus flyby orbit in this case is approximately 11,000 km.

3) The Venus flyby can be external or internal with respect to the sun.

4) For the re-eclipses, a two-impulse correction is necessary. The first correction completely corrects the time of flight to the probe orbit node and partially corrects the period, and the second correction completes the correction of the period. For the first eclipse, a one-impulse correction of flight time is sufficient. The first correction in both cases must be carried out 10 to 20 days after Venus flyby. The second correction must be made at the moment of probe eclipse.

5) The maximum correction velocity, with an allowed Venus flyby deviation of 1000 km, is 300 m/sec for the correction of both flight time and period, and 150 m/sec to 170 m/sec for the correction of flight time only.

6) The periodicity of launch dates is the same as for the flight to Venus. The launch window is $\sim$20 days.

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